Dynamic Assignment with Limited Commitment

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December 24, 2020

Introduction

- *P*rincipal dynamically allocates object(s) to *A*gent
- A's private value evolves over time
- P can not use money, cannot commit to future allocation
- P wants to allocate only when agent type is high

Introduction

- $\underline{P}$rincipal dynamically allocates object(s) to $\underline{A}$gent
- A's private value evolves over time
- P can not use money, cannot commit to future allocation
- P wants to allocate only when agent type is high

Not necessarily physical resources

- P: firm owner; A: expert
- A's private information: project return
- P is more conscious than A

This Paper

is not about...

- repeated cheap talk/ delegation
- full commitment
- durable good selling
- independent/constant types
- limit case $\delta \to 1$

is about...

- general mechanisms
- commitment to today
- allocation w.o. money
- persistent types
- fixed discount δ

Leading Example

- 1 *P*rincipal, 1 *A*gent, 1 unit, $T = 2$, no discount
- *A*'s private payoff type: $\theta_1 = l$ or h with equal probability, $l = 1$, $h = 3$; $\theta_2 = \theta_1$ with probability $\frac{3}{4}$
- *P* can supply at cost $c = \frac{7}{4} < E(\theta) = 2$
- Payoff in case of allocation: θ_t for *A*, $(\theta_t - c)$ for *P*

Leading Example

What can P obtain?

- No contracting: immediate allocation
- Contracting with full commitment?
- Contracting without inter-temporal commitment?

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The 2-period problem is essentially static

- No information can be elicited in period 2
- Only period 1 report matters

Commitment Solution (Guo and Horner, 2018)

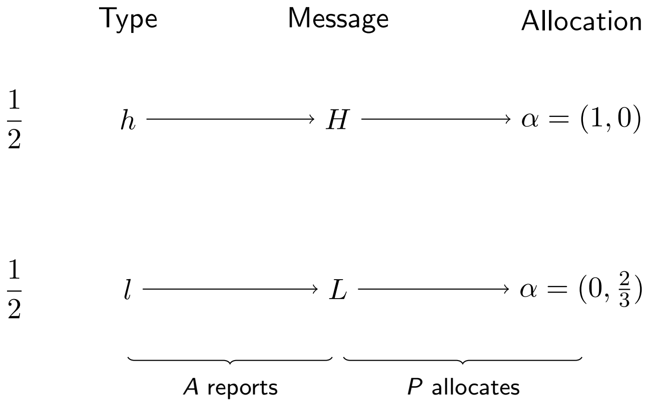

Figure 1: Optimal direct mechanism with commitment

Role of Commitment

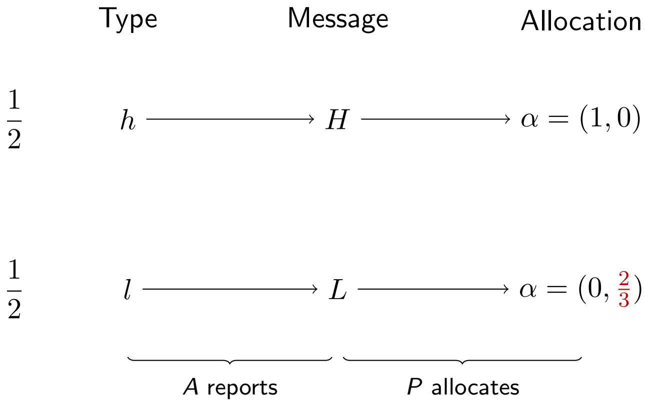

Figure 2: Optimal direct mechanism with commitment

Role of Commitment

	Type	Message	Allocation
$\frac{1}{2}$	$h \longrightarrow$	$H \longrightarrow$	$\alpha = (1, 0)$
		reward L with the 2nd chance	
$\frac{1}{2}$	$l \longrightarrow$	$L \longrightarrow$	$\alpha = (0, \frac{2}{3})$

$\underbrace{\qquad\qquad}_{A \text{ reports}}$ $\underbrace{\qquad\qquad}_{P \text{ allocates}}$

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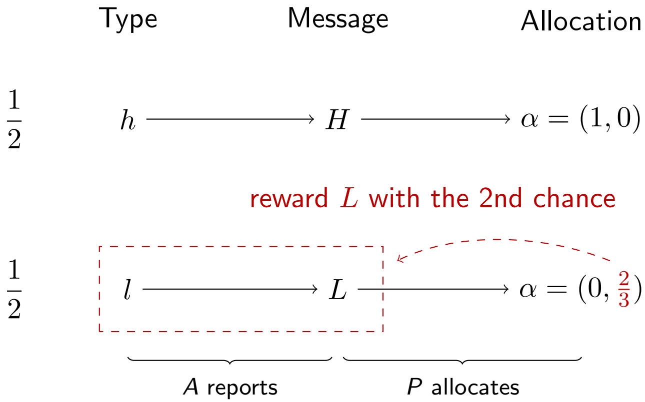

Figure 2: Optimal direct mechanism with commitment

This Paper: Limited Inter-temporal Commitment

P's Sequential Rationality

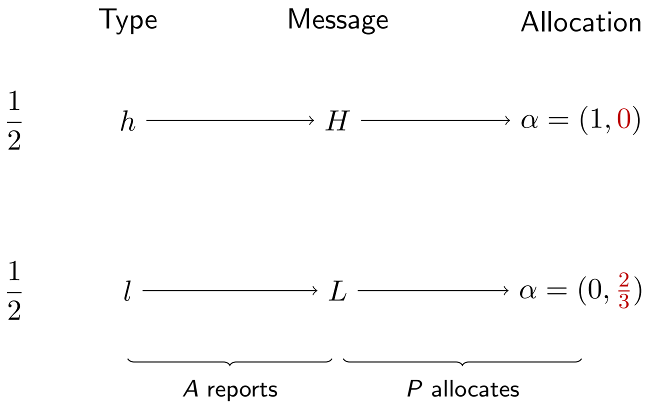

Figure 3: Optimal mechanism fails without commitment

	Type	Message	Allocation
$\frac{1}{2}$	$h \longrightarrow$	$H \longrightarrow$	$\alpha = (1, 0)$
			$\alpha_2 > 0$ only if $\mu_2 \geq \frac{3}{8}$, but $\mu_2 = \frac{1}{4}$
$\frac{1}{2}$	$l \longrightarrow$	$L \longrightarrow$	$\alpha = (0, \frac{2}{3})$
	A reports	P allocates	

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This Paper: Limited Inter-temporal Commitment

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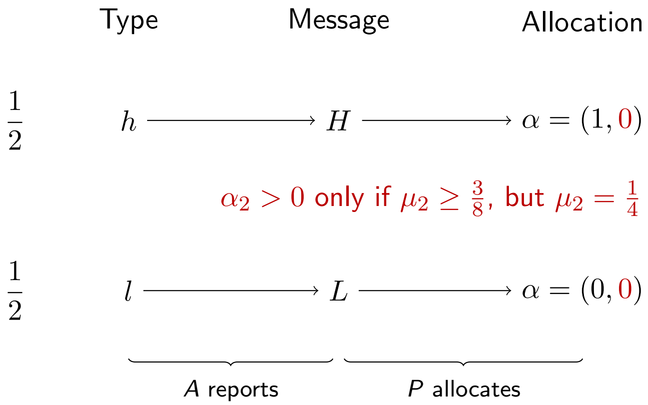

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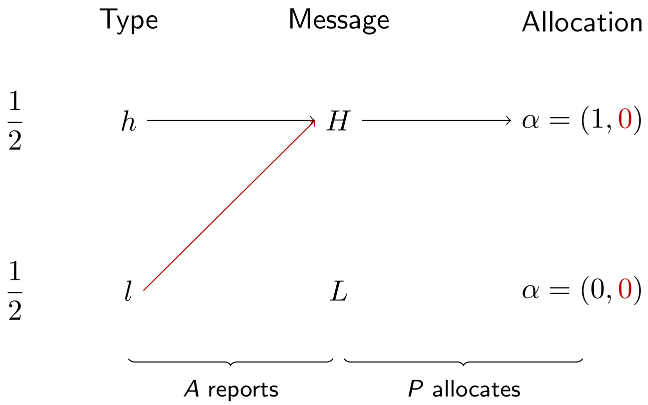


Figure 3: Optimal mechanism fails without commitment

What Can P Do?

Type	Input	Output
$\frac{1}{2}$ h	h	H
$\frac{1}{2}$ l	l	L

Figure 4: Optimal 2-signal communication device in period 1

An Improvement Through Noisy Communication

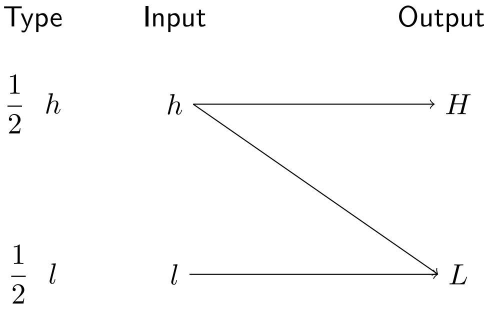
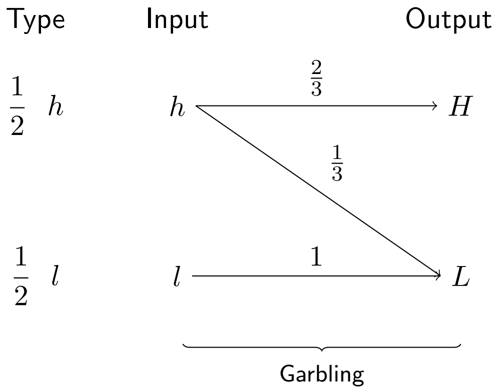


Figure 4: Optimal 2-signal communication device in period 1

An Improvement Through Noisy Communication



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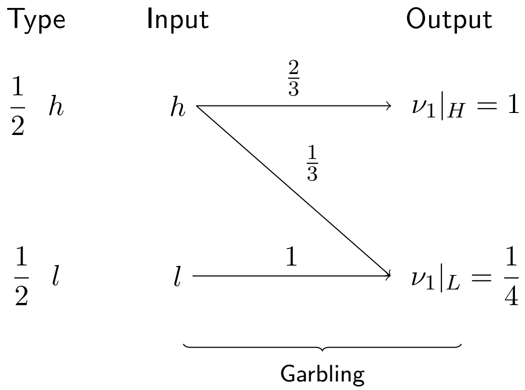

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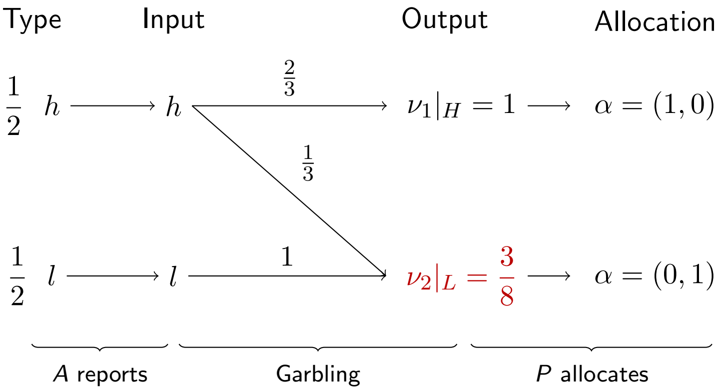
An Improvement Through Noisy Communication



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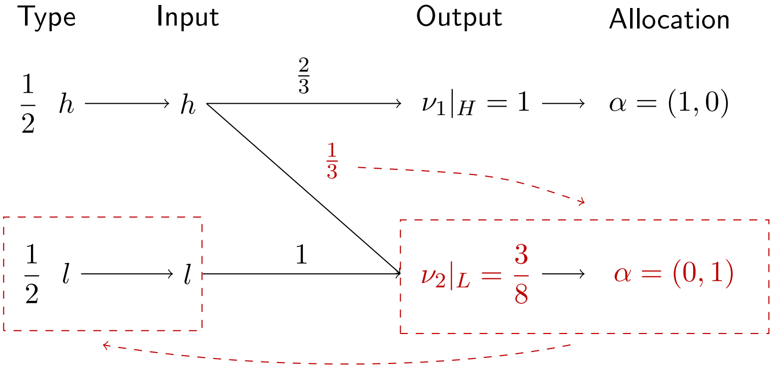


Figure 4: Optimal 2-signal communication device in period 1

The Optimal Noisy Communication

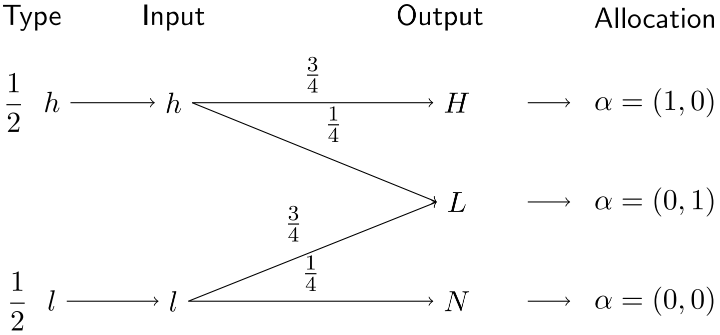
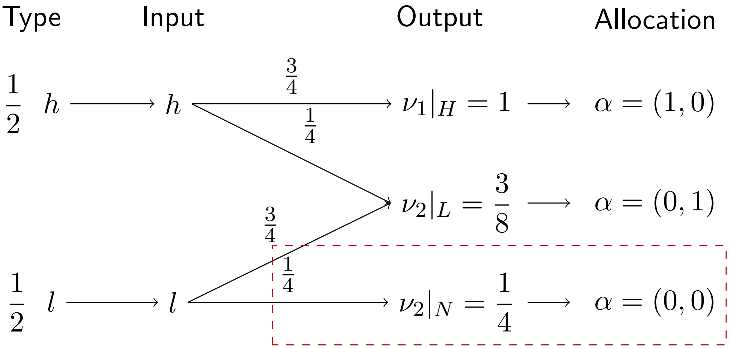


Figure 5: Optimal communication device in period 1

The Optimal Noisy Communication



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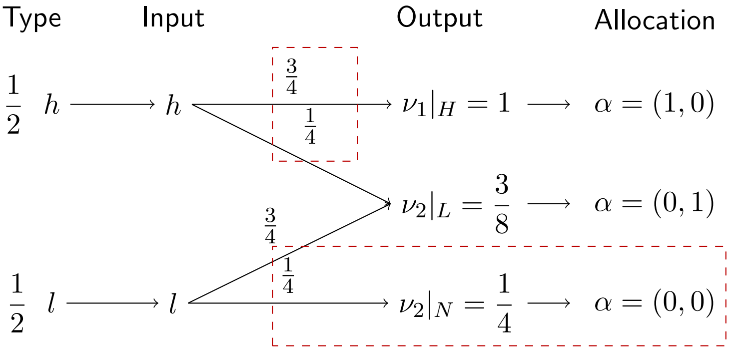

Figure 5: Optimal communication device in period 1

MODEL

Model

Environment

- 1 *P*rincipal, 1 *A*gent, 1 item, $2 \leq T \leq \infty$, common discount δ
- *A*'s value: $\theta_t \in \{l, h\}$, $P(v_1 = h) = \frac{1}{2}$, $P(h|h) = P(l|l) = \rho \geq \frac{1}{2}$
- *P* supplies at a constant cost: $c \in (l, h)$
- *P* cannot commit to the future allocation
- Payoff of *A*: $\sum_t \delta^{t-1} a_t \theta_t$, payoff of *P*: $\sum_t \delta^{t-1} a_t (\theta_t - c)$

Model

Mechanism

- In each period, *P* proposes a mechanism $((R_t, S_t, \beta_t), \alpha_t)$
- *A* chooses a report $r_t \in R_t$
- A random signal $s_t \in S_t$ is generated according to $\beta_t : R_t \to \Delta(S_t)$
- The allocation is realized $\alpha_t : S_t \to \Delta(A_t)$ where $A_t = \{0, 1\}$

Technical Approach (Doval & Skreta, 2020)

Classical revelation principle fails without full commitment

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Classical revelation principle fails without full commitment

Revelation principle with full commitment

Type	Input	Output	Allocation
Θ_t $\longrightarrow$	R_t $\longrightarrow$	S_t $\longrightarrow$	A_t

Figure 6: A general stage mechanism

Technical Approach (Doval & Skreta, 2020)

Classical revelation principle fails without full commitment

Revelation principle with full commitment

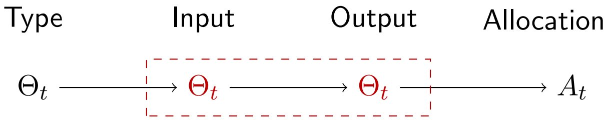

Figure 7: A direct stage mechanism

Technical Approach (Doval & Skreta, 2020)

Classical revelation principle fails without full commitment

Revelation principle with limited commitment

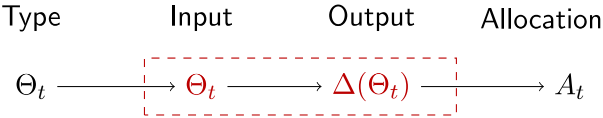

Figure 8: A canonical stage mechanism

Technical Approach (Doval & Skreta, 2020)

Classical revelation principle fails without full commitment

Revelation principle with limited commitment

Type Input Output Allocation

$$\Theta_t \longrightarrow \Theta_t \longrightarrow \Delta(\Theta_t) \longrightarrow A_t$$

Figure 8: A canonical stage mechanism

Dynamic mechanism $\Rightarrow$ $\underbrace{\text{self-persuasion}}_{\text{constrained by sincere report}}$ $+$ stage mechanism

RESULTS T=2

Notation

Let $\mu_t, \nu_t \in \Delta(\Theta)$ denote P's belief about θ_t

- Let μ_t denote P's belief at the beginning of t
- Let ν_t denote P's belief at the end of t
- When $T = 2$, $\underbrace{\nu_2 = \mu_2}_{\text{pooling}} \underbrace{= \nu_1 \rho + (1 - \nu_1)(1 - \rho)}_{\text{evolution}}$

Let $w_t(\nu_t)$ denote P's conditional stage payoff

- $w_1(\nu_1) = \alpha_1(\nu_1)(E_{\nu_1}(\theta_1) - c)$
- $w_2(\nu_2) = (1 - \alpha_1(\nu_1))\alpha_2(\nu_2)(E_{\nu_2}(\theta_2) - c)$

Second Period

In period 2, the mechanism selection only depends on ν_2:

$$\alpha_2(\nu_2) = 1 \text{ iff } E_{\nu_2}(\theta_2) - c \geq 0$$
$$\text{iff } \nu_2 \geq \frac{c-l}{h-l}$$
$$\text{iff } \nu_1 \geq \frac{1}{2\rho - 1}\left(\frac{c-l}{h-l} - 1 + \rho\right)$$

Optimal Mechanism $T = 2$

In the first period, the principal designs a contract (β, α) to maximize her expected payoff:

$$\max_{(\beta,\alpha)} \sum_{\nu_1 \in \Delta(\Theta)} \left(\sum_{\theta_1 \in \{l,h\}} \mu(\theta_1)\beta(\nu_1|\theta_1) \right) [w_1(\nu_1) + \delta w_2(\nu_1)]$$

$$[IC_{\theta_1}] \quad \sum_{\nu_1 \in \Delta(\Theta)} \left[\beta(\nu_1|\theta_1) - \beta(\nu_1|\hat{\theta}_1) \right] [\alpha_1\theta_1 + \delta(1-\alpha_1)\alpha_2 E(\theta_2|\theta_1)] \geq 0$$

$$[BC_{\nu_1}] \quad \nu_1 \left(\sum_{\theta_1 \in \{l,h\}} \mu_1(\theta_1)\beta(\nu_1|\theta_1) \right) = \mu_1(h)\beta(\nu_1|h)$$

How to solve the constrained persuasion? cbp

Optimal Mechanism $T = 2$

In the first period, the principal designs a contract (τ, α) to maximize her expected payoff:

$$\max_{(\tau,\alpha)} \sum_{\nu_1 \in \Delta(\Theta)} \tau(\nu_1) \left[w_1(\nu_1) + \delta w_2(\nu_1) \right]$$

$$[IC_l] \quad \sum_{\nu_1 \in \Delta(\Theta)} \tau(\nu_1) \frac{\mu_1 - \nu_1}{\mu_1(1-\mu_1)} \left[\alpha_1 \theta_1 + \delta(1-\alpha_1)\alpha_2 E(\theta_2|\theta_1) \right] = 0$$

$$[BC_{\nu_1}] \quad \sum_{\nu_1 \in \Delta(\Theta)} \tau(\nu_1)\nu_1 = \mu_1$$

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Optimal Mechanism $T = 2$

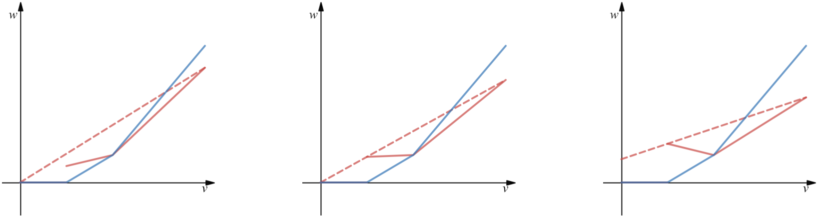


Figure 9: The solid blue line depicts $w(\nu_1)$, while the solid and dashed red lines depict $\mathcal{L}(\nu_1, \eta) = w(\nu_1) + \eta IC(\nu_1)$ and its concavification

Optimal Mechanism $T = 2$

Proposition

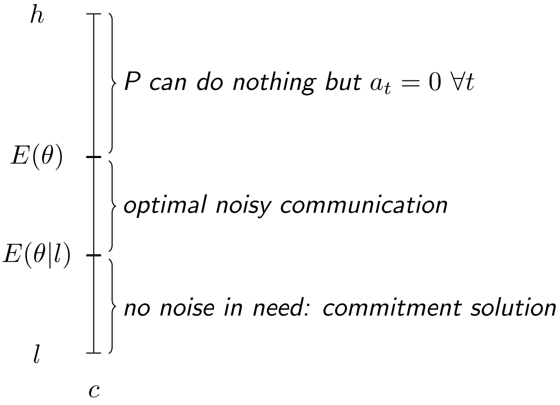
Suppose $\delta \geq \frac{l}{E(\theta|l)}$*, the optimal mechanism is as follows:*

- *When* $c \geq E(\theta)$, $S_1 = \{\mu_1\}$, $\beta_1(\mu_1|h) = \beta_1(\mu_1|l) = 1$, $\alpha_1(\mu_1) = \alpha_2(\mu_1) = 0$.
- *When* $E(\theta|l) \leq c < E(\theta)$, $S_1 = \{0, \bar{\nu}, 1\}$, $\beta_1(1|h) = 1 - \beta_1(\bar{\nu}|h) = \frac{\delta(1-2\bar{\nu})E(\theta|l)}{\bar{\nu}l+\delta(1-2\bar{\nu})E(\theta|l)}$, $\beta_1(\bar{\nu}|l) = 1 - \beta_1(0|l) = \frac{(1-\bar{\nu})l}{\bar{\nu}l+\delta(1-2\bar{\nu})E(\theta|l)}$, $\alpha_1(1) = 1$, $\alpha_1(\bar{\nu}) = \alpha_1(0) = 0$, $\alpha_2(\bar{\nu}) = 1$, $\alpha_2(0) = 0$.
- *When* $c < E(\theta|l)$, $S_1 = \{0, 1\}$, $\beta_1(1|h) = \beta_1(0|l) = 1$, $\alpha_1(1) = 1$, $\alpha_1(0) = 0$ *and* $\alpha_2(0) = 1$.

Optimal Mechanism $T = 2$

Proposition

When $\delta \geq \frac{l}{E(\theta|l)}$, the optimum is as follows:



Remark 1: Menu Implementation

The optimum is not implementable by a menu of contracts

- Menu implementation: agent randomization/mixed strategy
- IC_h is slack: h should generate 2 messages but has no incentive to randomize

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The optimum is not implementable by a menu of contracts

- Menu implementation: agent randomization/mixed strategy
- IC_h is slack: h should generate 2 messages but has no incentive to randomize

Implication: an explicit garbling device is NECESSARY

In contrast:

- With money: transfer fills payoff gap (Doval and Skreta, 20)
- With commitment: no need to distort h (Guo and Horner, 18)

Remark 2: Capacity Restriction

What if P can allocate 1 unit in each period?

Lemma

If $T < \infty$, $A_t = \{0, 1\}$ $\forall t \leq T$, P cannot improve upon the default action.

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What if P can allocate 1 unit in each period?

Lemma

If $T < \infty$, $A_t = \{0, 1\}$ $\forall t \leq T$, P cannot improve upon the default action.

Proof.

- $t = T$: no information, allocation based on belief;
- $t = T - 1$: low type deviates to report high.

□

Remark 2: Capacity Restriction

Two commitment issues

- Punishing H: it is not too tempting to mimic h
- Rewarding L: it is worthwhile to admit l

Remark 2: Capacity Restriction

Two commitment issues

- Punishing H: it is not too tempting to mimic h

  requires exogenous capacity restriction

- Rewarding L: it is worthwhile to admit l

  can be overcome by noisy communication

Extensions

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More than 2 periods? More than 2 types?

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Optimal Belief Path $T = 2$

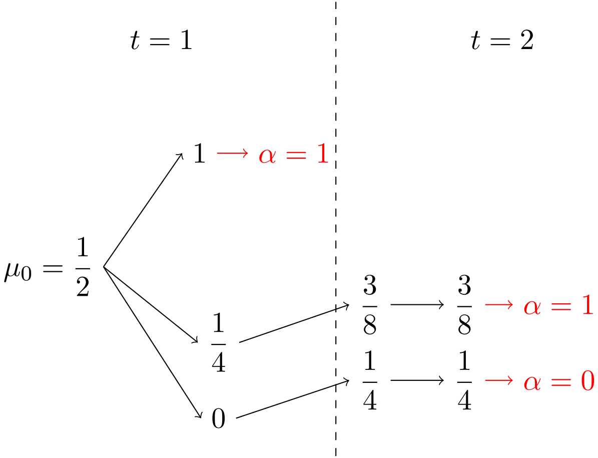

Figure 10: The principal's belief path at optimum

Optimal Belief Path $T < \infty$

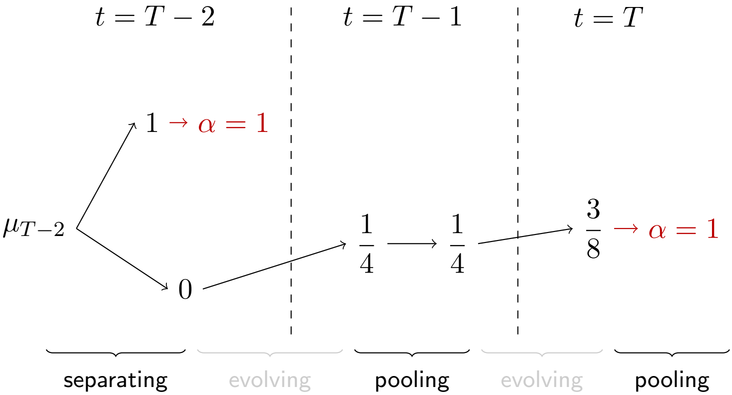

Figure 11: The optimal belief path when $3 \leq T < \infty$, $\rho = \frac{3}{4}$, $c = \frac{7}{4}$

Optimal Belief Path $T < \infty$

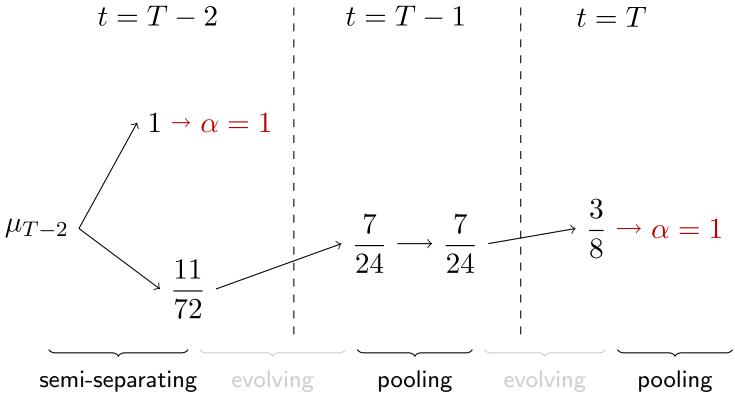

Figure 12: The optimal belief path when $3 \leq T < \infty$, $\rho = \frac{4}{5}$, $c = \frac{7}{4}$

Optimal Belief Path $T < \infty$

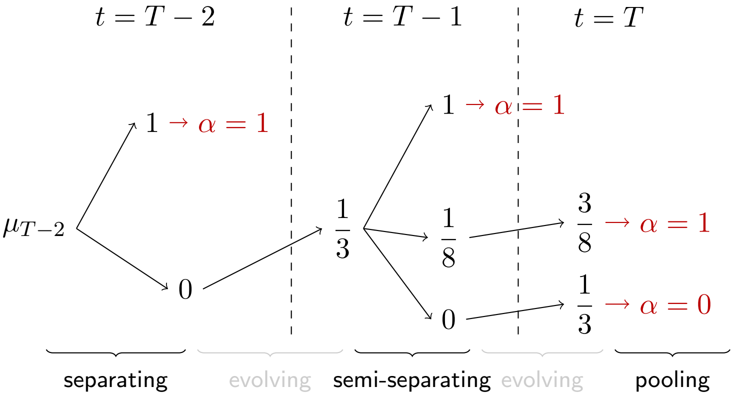

Figure 13: The optimal belief path when $3 \leq T < \infty$, $\rho = \frac{2}{3}$, $c = \frac{7}{4}$

Optimal Belief Path $T = \infty$

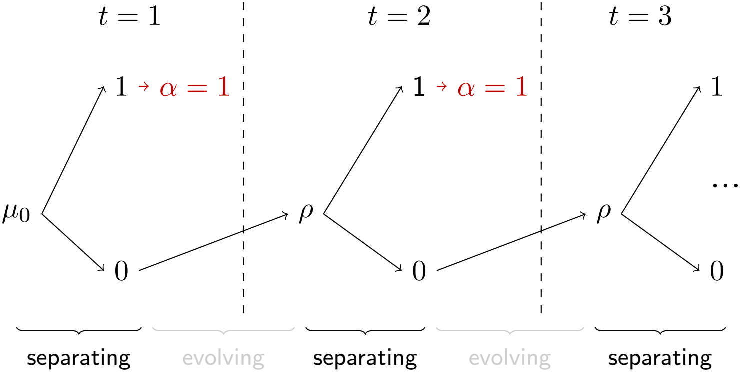

Figure 14: The optimal belief path when $T = \infty$

Extensions

More than 2 periods? More than 2 types?

Bad images wash out through mean-reversion

Distortion is back-loaded as much as possible

Extensions

More than 2 periods? More than 2 types?

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Interval Type Space $\Theta = [0, 1]$

Model:

- $T = 2$, $c < 0.5$, $\delta = 1$, total capacity is 1
- $\theta_1 \sim U[0, 1]$, $\theta_2 = \theta_1$ w.p. λ and redrawn w.p. $1 - \lambda$
- Default: allocate immediately

Interval Type Space $\Theta = [0, 1]$

Model:

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- $\theta_1 \sim U[0, 1]$, $\theta_2 = \theta_1$ w.p. λ and redrawn w.p. $1 - \lambda$
- Default: allocate immediately

Self-persuasion with continuous type

- Period 2: only the perceived mean $E(\theta_2|\beta, r_1)$ matters
- Period 1: $E(\theta_2|\beta, r_1) = \lambda E(\theta_1|\beta, r_1) + 0.5(1 - \lambda)$
- Only posterior mean matters: output message reduced to 1-dimensional

Interval Type Space $\Theta = [0,1]$

Optimal Mechanism with medium cost:

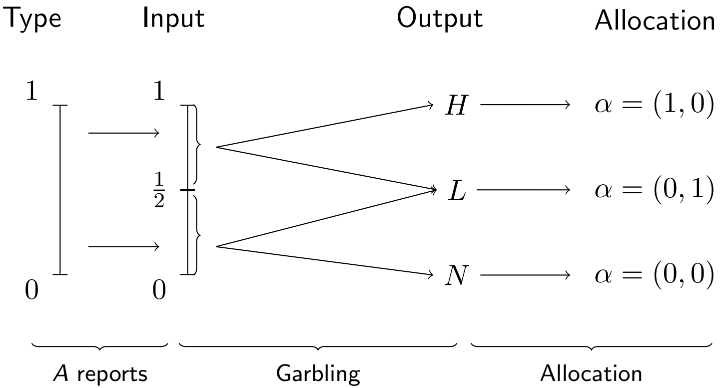

Figure 15: Optimal mechanism with continuous type

Extensions

More than 2 periods? More than 2 types?

Simple mechanism similar to binary type

Open Questions

$T < \infty$, $K < T$ units

$T = \infty$, 1 unit each period

Literature

Mechanism design w/o money: Guo and Horner (2018), Balseiro et al. (2018), Lipnowski and Ramos (2019), Chen (2019), etc.

Revelation principle w/o commitment: Myerson (1982), Forges (1986), Bester and Strausz (2001, 2007), Doval and Skreta (2020a, 2020b), Xu and Xu (2020), etc.

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Revelation principle w/o commitment: Myerson (1982), Forges (1986), Bester and Strausz (2001, 2007), Doval and Skreta (2020a, 2020b), Xu and Xu (2020), etc.

Noise improves information transmission: Blume et al. (2007), Ivanov (2014), Salamanca (2016), Blume et al. (2019), etc.

Rebuilding commitment through semi-separation: Hosios and Peters (1989), Dewatripont and Maskin (1995), Nosal and Ordonez, (2016); Carrillo and Mariotti (2000), Brocas et al. (2004); Ball (2020); Whitmeyer (2020), etc.

Conclusion

- Dynamic allocation/delegation in a changing world
- Tension between incentive provision and lack of commitment
- Commitment partially rebuilt by noisy communication
- Key insight: commit by manipulating future belief
- The noisy channel is necessary for implementation

Thanks!

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How to solve Bayesian Persuasion

For any primary BP problem (w, μ), the value of the optimal signal structure is obtained by

$$\max_{(\tau,\nu)} \sum_{\nu} \tau(\nu) w(\nu)$$

$$\text{s.t.} \quad \sum_{supp(\tau)} \tau(\nu)\nu = \mu$$

Geometrically, $supp(\tau)$ and τ are solved by concavifying $w(\nu)$

Back

How to solve Constrained Bayesian Persuasion

For any primary c-BP problem (w, μ), the value of the optimal signal structure is obtained by

$$\max_{(\tau,\nu)} \sum_{\nu} \tau(\nu) w(\nu)$$

$$\text{s.t.} \quad \sum_{supp(\tau)} \tau(\nu)\nu = \mu$$

$$\sum_{supp(\tau)} \tau(\nu) g(\nu) = 0$$

Geometrically, $supp(\tau)$ and τ are solved by concavifying Lagrangian $w(\nu) + \eta g(\nu)$ and η is pinned down by $\sum_{supp(\tau)} \tau(\nu) g(\nu) = 0$

Back